Unit - IV

Analysis of Variance

Experiment:-

An experiment is just a test or series of test. experiments are performed in all engineering disciplines are an important part of the way, we learn about how systems work.

Experimental units:-

The object upon which the measurements are taken are called experimental units.

Basic principles in the design of experiments:- (6).

There are 3 basic principles of experimental design, they are

* Randomization
* Replication
* Local control.

Adv of CRD: (10)

* It is easy to layout the design
* It allows for more flexibility, simplicity, validity.

Randomization:- (8).

A set of objects is said to be randomized when they are arranged in random order.

(11) → ANOVA - LSD.

(12) LSD - RBD.

Adv. of Latin Square:- (9).

* with two-way classification Latin square control more of the variations than the CRD and RBD.
* It is simple, but compared to RBD, LSD is complicated
* The analysis remains simple even with missing data.

Replication:- (4). Adv * reduce the experimental error
* increasing the efficiency of design.

* It is the independent execution of an experiment more than once.

* It is necessary to increase the accuracy of estimates of the treatment effects.

'repitition of treatments under investigation'.

Local control:-

* This includes techniques such as grouping, blocking and balancing of experimental units used in the experimental design.

Complete block designs:-

* Completely randomized Design (CRD).

* Randomized Block Design (RBD)

* Latin square Design. (LSD)

Analysis of variance (ANOVA):- (1)

'ANOVA' is an arithmetical procedure, used to express the total variation of data as the sum of its non-negative components.

Assumptions made in ANOVA:- (5).

* The observation are independent.

* Parent population from which Observations are taken in normal. population

(2)

Uses of ANOVA:-

* To test the homogenity of several means.
technique is now frequently applied

One way classification:-

Completely Randomized Design (CRD):-

This is the simplest of all designs based on the principles of randomization and ______.

Applications of CRD:-

* It is most useful in laboratory technique and methodological studies.
* This design is flexible, any no. of treatments can be used.

Disadvantages of CRD:-

This design is less efficient.

Working Procedure:-

Step 1:

Assume H_0: There is no significant difference between the treatments.

H_1: There is significant difference between the treatments.

Step 2:

Find the no. of observations N.

Step 3:-

Find the total value of all the observation "T" $T = \Sigma x_1 + \Sigma x_2 + \ldots$

step 4:

Calculate the correction factor $CF = \frac{T^2}{N}$

step 5:

calculate the total sum of square

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \cdots - \frac{T^2}{N}.$$

step 6:

calculate the column sum of squares

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \cdots\cdots - \frac{T^2}{N}.$$

where, N is the no. of elements in each column or row.

step 7:

calculate error sum of squares

$$SSE = TSS - SSC$$

Step 8:-

Prepare the "ANOVA" table to calculate F-ratio. ③ Two way → ④.

ANOVA TABLE:-

Source of Variation	sum of squares (SS)	d.f	Mean square	Variance ratio	Table Value
Between columns	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F_c = \frac{MSE}{MSC}$ or	
Error	SSE	$N-c$	$MSE = \frac{SSE}{N-c}$	$F_c = \frac{MSC}{MSE}$	
Total	TSS				

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Step 9:

Write the conclusion.

Problems:

1. There are three main brands of certain powder. A set of 120 sample values is examined and found to be allocated among 4 groups A, B, C, D and three brands I, II, III as shown here under.

Brands	Groups			
	A	B	C	D
I	0	4	8	15
II	5	8	13	6
III	8	19	11	13.

Is there any significant difference in brands preference? Assume at 5% level

Sol:-

Step 1:-

Assume.

H_0 : There is no significant difference in brands.

H_1 : There is significant difference in brands.

Brands	Groups				Total	X_1^2	X_2^2	X_3^2	X_4^2
	A (X_1)	B (X_2)	C (X_3)	D (X_4)					
I (Y_1)	0	4	8	15	27	0	16	64	225
II (Y_2)	5	8	13	6	32	25	64	169	36
III (Y_3)	8	19	11	13	51	64	361	121	169
Total	13	31	32	34	110	89 $\cancel{441}$	441 $\cancel{35}$	354	430

Step 2:

No. of observation $= N = 12$

Step 3:

$T = 110.$

Step 4:

$$\frac{T^2}{N} = \frac{(110)^2}{12} = 1008.3.$$

Step 5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$$

$$TSS = 89 + 441 + 354 + 430 - 1008.3$$

$$TSS = 305.7.$$

Step 6:

$$SSR = \frac{(\Sigma Y_1)^2}{N_1} + \frac{(\Sigma Y_2)^2}{N_1} + \frac{(\Sigma Y_3)^2}{N_1} - \frac{T^2}{N}$$

$N_1 \rightarrow$ No. of elements in each row.

$$= \frac{(27)^2}{4} + \frac{(32)^2}{4} + \frac{(51)^2}{4} - 1008.3$$

$$SSR = 80.2$$

Step 7:

$SSE = TSS - SSR$

$= 305.7 - 80.2$

$= 225.5.$

Step 8:

ANOVA TABLE:-

Source of Variation	Sum of squares	Degrees of freedom	Mean squares	Variance ratio	Table value at 5% level
Between rows	$SSR = 80.2$	$r-1 = 3-1$, $r = 2$	$MSR = \frac{SSR}{r-1} = \frac{80.2}{2} = 40.1$	$F_{cal} = \frac{MSR}{MSE} = \frac{40.1}{25.05} = 1.601$	$F_{tab} = (2,$ $= 4.26$
Error	$SSE = 225.5$	$N-r = 12-3$, $N-r = 9$	$MSE = \frac{SSE}{N-r} = \frac{225.50}{9} = 26.05$		

Step 9:-

Conclusion:

calculated $F_R = 1.6$, Table $F_R = 4.26$.

Here cal $F_R <$ Tab F_R.

$\therefore$ We accept H_0.

There is no significance difference between brands.

2. The following are the no. of mistakes made in 5 successive days of 4 technician working for a photography Laboratory.

Techinician 1	Technician 2	Technician 3	Technician 4
X_1	X_2	X_3	X_4
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Test at the LOS $\alpha = 0.01$, whether the differences among the 4 sample means can be attributed to chance.

Sol:-

Step 1:

H_0 = There is no significant difference between the technician

H_1 = There is significant difference between the technician.

Origin 10.

X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
-4	4	0	-1	-1	16	16	0	1
4	-1	2	2	7	16	1	4	4
0	2	-3	-2	-3	0	4	9	4
-2	0	5	0	3	4	0	25	0
1	4	1	1	7	1	16	1	1
-1	9	5	0	13	37	37	39	10

Step 2:

No. of observation $N = 20$.

Step 3:

$T = 13$.

Step 4:

$$\frac{T^2}{N} = \frac{(13)^2}{20} = 8.45.$$

Step 5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$$

$$= 37 + 37 + 39 + 10 - 8.45$$

$TSS = 114.55.$

Step 6:

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} + \frac{0^2}{5} - 8.45$$

$= 12.95.$

Step 7:-

$SSE = TSS - SSC$

$TSS = 114.55$

$SSC = 12.95.$

$SSE = 114.55 - 12.95.$

$SSE = 101.6.$

step 8:-

ANOVA TABLE:-

Source of variation	Sum of squares	Degrees of freedom	Mean squares	Variance ratio	Table value
Between columns	SSC = 12.95	c−1 = 4−1 = 3	$MSC = \frac{SSC}{c-1} = \frac{12.95}{4-1} = 4.32$	$F_c = \frac{MSE}{MSC} = \frac{6.35}{4.32} = 1.471$ $\left[\frac{MSC}{MSE} < 1\right]$	$F_c(16, 3) = 5.29$.
Error	SSE = 101.6	N−c = 20−4 = 16	$MSE = \frac{SSE}{N-c} = \frac{101.6}{20.4} = 6.35$		

step 9:

Conclusion:-

$F_{cal} < F_{Tab}$

So we accept H_0

∴ There is no significant ~~sign~~. difference between technicians.

3. The following table shows the lives in hours of 4 brands of electric lamps.

Brand A	1610	1610	1650	1680	1700	1720	1800	—
Brand B	1580	1640	1640	1700	1750	—	—	—
Brand C	1460	1550	1600	1620	1640	1660	1740	1820
Brand D	1510	1520	1530	1570	1600	1680	—	—

Perform an ANOVA test, the homogenity of mean lives of the four brands of lamps.

Sol:-

Step 1:

H_0: There's no significande difference in brands.

H_1: There's significance difference in brands.

Step 2:

No. of. observations $N = 26$.

Origin: Subtract 1600 and divide by 10.

x_1	x_2	x_3	x_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
1	-2	-14	-9	-24	1	4	196	81
1	4	-5	-8	-8	1	16	25	64
5	4	0	-7	2	25	16	0	49
8	10	2	-3	17	64	100	4	9
10	15	4	0	29	100	225	16	0
12	-	6	8	26	144	-	36	64
20	-	14	-	34	400	-	196	-
-	-	22	-	22	-	-	484	-
Tot: $\Sigma x_1 = 57$	$\Sigma x_2 = 31$	$\Sigma x_3 = 29$	$\Sigma x_4 = -19$	$T = 98$	$\Sigma x_1^2 = \cancel{788}$ 735	$\Sigma x_2^2 = 361$	$\Sigma x_3^2 = 957$	$\Sigma x_4^2 = 267$

Step 3:

Correction or Error factor

$$\frac{T^2}{N} = \frac{(98)^2}{26} = 369.38.$$

step 4:-

Total sum of square:-

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$$

$$= 735 + 361 + 957 + 267 - 369.38$$

$$TSS = 1950.62$$

step 5:-

SSC - Sum of squares of column.

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{(57)^2}{7} + \frac{(31)^2}{5} + \frac{(29)^2}{8} + \frac{(-19)^2}{6} - 369.38$$

$$= 464.14 + 192.2 + 105.12 + 60.16 - 369.38.$$

$$SSC = 452.24$$

step 6:-

Sum of squares of Error

$$SSE = TSS - SSC$$

$$= 1950.62 - 452.24$$

$$SSE = 1498.38$$

$\therefore$ $TSS = 1950.62$

$SSC = 452.24$

$SSE = 1498.38$

Step 7: ANOVA TABLE:-

Source of Variation	Sum of Squares	DOF	Mean Squares	Variance ratio	Table value at 5% level
Between columns	SSC = 452.24	$c-1$ $=4-1$ $=3$	$MSC = \frac{SSC}{c-1}$ $= \frac{452.24}{3}$ $= 150.75$	$F_{cal} = \frac{MSC}{MSE}$ $= \frac{150.75}{68.11}$ $= 2.21$	$F_{tab(3,22)}$ $= 3.05$
Error	SSE = 1498.37	$N-c$ $=26-4$ $=22$	$MSE = \frac{SSE}{N-c}$ $= \frac{1498.37}{22}$ $= 68.11$		

Step 8:-

conclusion:-

Here $F_{cal} < F_{tab}$

$\therefore$ We accept H_0.

There is no significant difference b/w brands.

4. H/w.

A completely randomised design experiments with 10 plots and 3 treatments gave the following results:-

Plot No:	1	2	3	4	5	6	7	8	9	10
Treat-ment:	A	B	C	A	C	C	A	B	A	B
Yield:	5	4	3	7	5	1	3	4	1	7

Analyze the results for treatment effect.

Sol:-

A	5	7	3	1
B	4	4	7	
C	3	5	1	

Step 1:

H_0: There's no significante difference between experiments.

H_1: There's significant difference between experiments.

X_1	X_2	X_3	Total	X_1^2	X_2^2	X_3^2
5	4	3	12	25	16	9
7	4	5	16	49	16	25
3	7	1	11	9	49	1
1	-	-	1	1	-	-
Σx_1 16	Σx_2 15	Σx_3 9	T = 40	$\Sigma x_1^2 =$ 84	$\Sigma x_2^2 =$ 81	$\Sigma x_3^2 =$ 35

Step 2:

No. of. observations. N = 10.

Step ~~4~~ 3:-

Grand Total T = 40.

Step 4:- correction factor

$$\frac{T^2}{N} = \frac{(40)^2}{10} = 160.$$

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step 5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \cancel{\Sigma x_4^2} - \frac{T^2}{N}$$

$$= 84 + 81 + 35 - 160$$

$$TSS = 40$$

step 6 :-

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(16)^2}{4} + \frac{15^2}{3} + \frac{9^2}{3} - 160.$$

$$= 64 + 75 + 27 - 160$$

$$SSC = 6$$

step 7 :-

$$SSE = TSS - SSC = 40 - 6 = 34.$$

Step 8 : ANOVA TABLE :-

Source of Variation	Sum of Squares	DoF	Mean squares	Variance ratio	Table Value at 5% level
Between Column	$SSC = 6$	$c-1$ $= 3-1$ $= 2$	$MSC = \frac{SSC}{c-1}$ $= \frac{6}{2}$ $= 3$	$F_{cal} = \frac{MSE}{MSC}$ $= \frac{4.86}{3}$ $= 1.62$	$F_{tab}(7,2)$ $\neq 1.9$ $= 19.3$
errors	$SSE = 34$	$N-c$ $= 10-3$ $= 7$	$MSE = \frac{SSE}{N-c}$ $= \frac{34}{7}$ $= 4.86$		

step 9 :- Conclusion :-

Here $F_{cal} < F_{tab}$

So, we accept H_0.

$\therefore$ There's no significant difference b/w experiments

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